Data-Driven Decomposition Techniques for Electromagnetic Induction Sensing of Underwater Munitions

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Denver, Colorado

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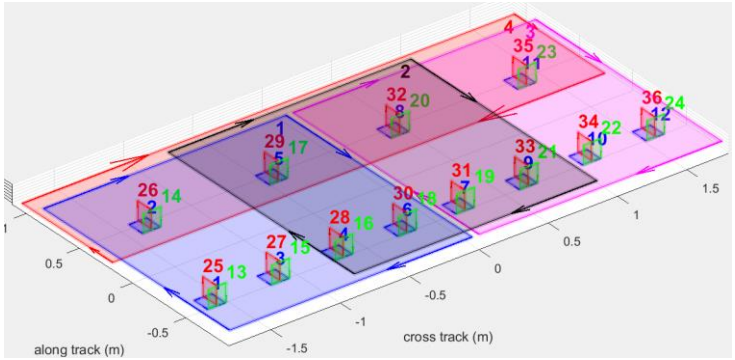
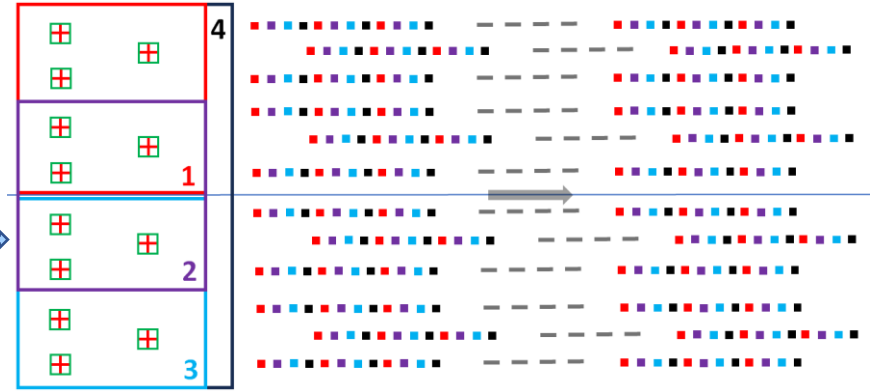
Marine Measurements

UltraTEM System: 4/3 Txs, 12 Rx cubes

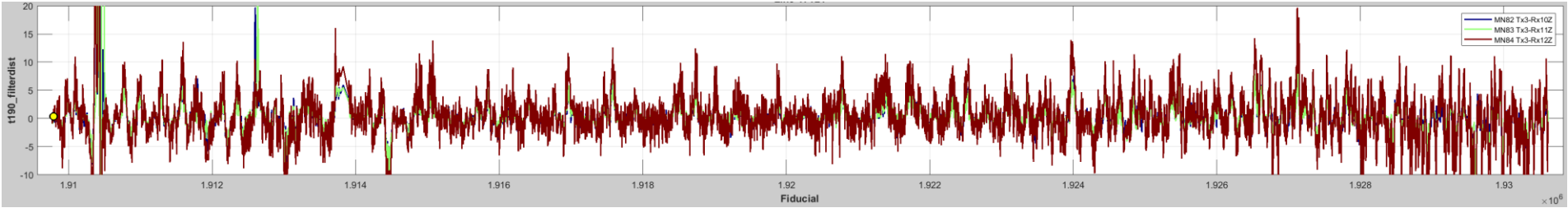


Firing and measuring

Dynamic data acquisition along a path



Response profile in a marine survey



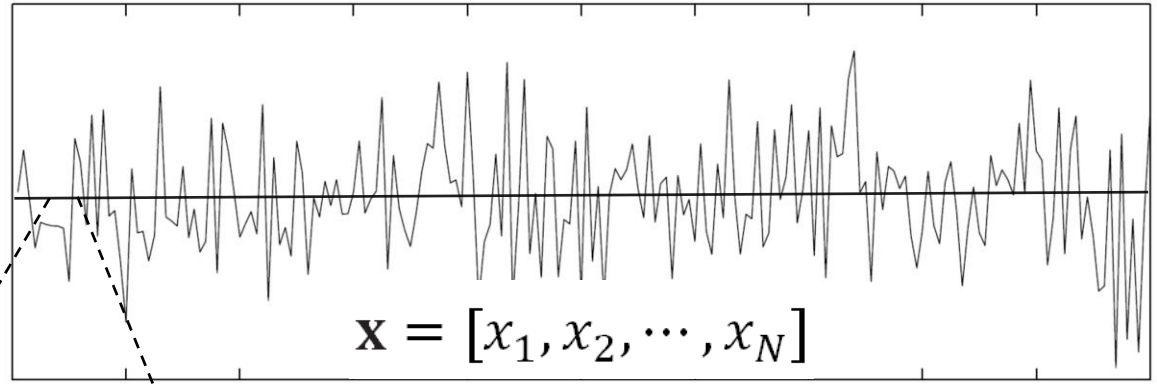
Outline

Data-Driven Decomposition Methods

a. Singular Spectrum Analysis (SSA)
b. Dynamic Mode Decomposition (DMD)
c. Experiments
d. Conclusions

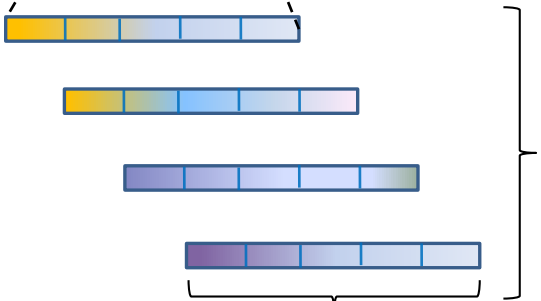
Singular Spectrum Analysis (SSA)

Embedding:



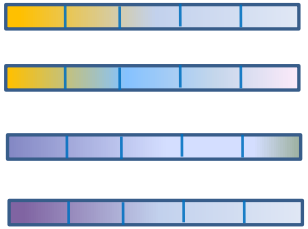
$$\begin{bmatrix} x_1 & \cdots & x_L \\ \vdots & \ddots & \vdots \\ x_K & \cdots & x_{K+L-1} \end{bmatrix}$$

Hankel matrix



K lagged vectors

Sliding window *L*



L x K matrix

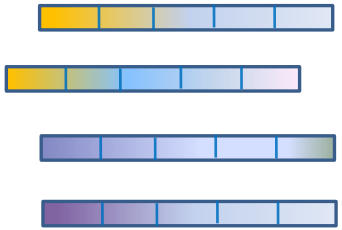
Singular Spectrum Analysis (SSA)

Procedure:

$$\mathbf{x} = [x_1, x_2, \cdots, x_N]$$

Hankel matrix H

$$\begin{bmatrix} x_1 & \cdots & x_L \\ \vdots & \ddots & \vdots \\ x_K & \cdots & x_{K+L-1} \end{bmatrix}$$



SVD

σ_1 $\mathbf{v}_1$ σ_2 $\mathbf{v}_2$

$\widehat{\mathrm{H}}_1$ $\widehat{\mathrm{H}}_2$ $\cdots$

$\mathbf{u}_1$ $\mathbf{u}_2$

Eelementary rank-one matrix

$$\widehat{\mathrm{H}}_i = \sigma_i \mathbf{u}_i \mathbf{v}_i^T, \qquad i = 1, 2, \cdots, r$$

Hankelizing

$$\widetilde{\mathrm{H}}_i = \mathcal{H}(\widehat{\mathrm{H}}_i)$$

Converting into $\tilde{\mathbf{x}}_i$

$$\tilde{\mathbf{x}}_i(1:K) = \widetilde{\mathrm{H}}_i(1:K, 1)$$

$$\tilde{\mathbf{x}}_i(K-1)+2:L) = \widetilde{\mathrm{H}}_i(K, 2:L)$$

Synthesizing by grouping

Dynamic Mode Decomposition (DMD)

1. Setup Linear mapping

- Split Hankel matrix H

$$\mathrm{H} = \begin{bmatrix} x_1 & \cdots & x_L \\ \vdots & \ddots & \vdots \\ x_K & \cdots & x_{K+L-1} \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & \vdots \\ \mathbf{h}_1 & \vdots & \mathbf{h}_L \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\mathrm{Y}_1 = \begin{bmatrix} \vdots & \vdots & \vdots \\ \mathbf{h}_1 & \vdots & \mathbf{h}_{L-1} \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\mathrm{Y}_2 = \begin{bmatrix} \vdots & \vdots & \vdots \\ \mathbf{h}_2 & \vdots & \mathbf{h}_L \\ \vdots & \vdots & \vdots \end{bmatrix}$$

- Propagation A

$$\mathrm{Y}_2 \approx \mathrm{A}\mathrm{Y}_1$$

2. Modal decomposition

- Perform the SVD of

$$\mathrm{Y}_1 = \mathrm{U}\Lambda\mathrm{V}^{\mathrm{T}}$$

- Construct the low-rank A

$$\tilde{\mathrm{A}} = \mathrm{U}_r^{\mathrm{T}}\mathrm{Y}_2\mathrm{V}_r\Lambda_r^{-1}$$

$$i = 1, 2, \cdots, r$$

- Compute eigen-pairs

$$\tilde{\mathrm{A}}\mathbf{q}_i = \gamma_i\mathbf{q}_i$$

3. Reconstruction

- Compute DMD mode

$$\phi_i = \mathrm{Y}_2\mathrm{V}_r\Lambda_r^{-1}\mathbf{q}_i$$

- Compute DMD frequency

$$\omega_i = \ln(\gamma_i)/\Delta t$$

- Set initial state

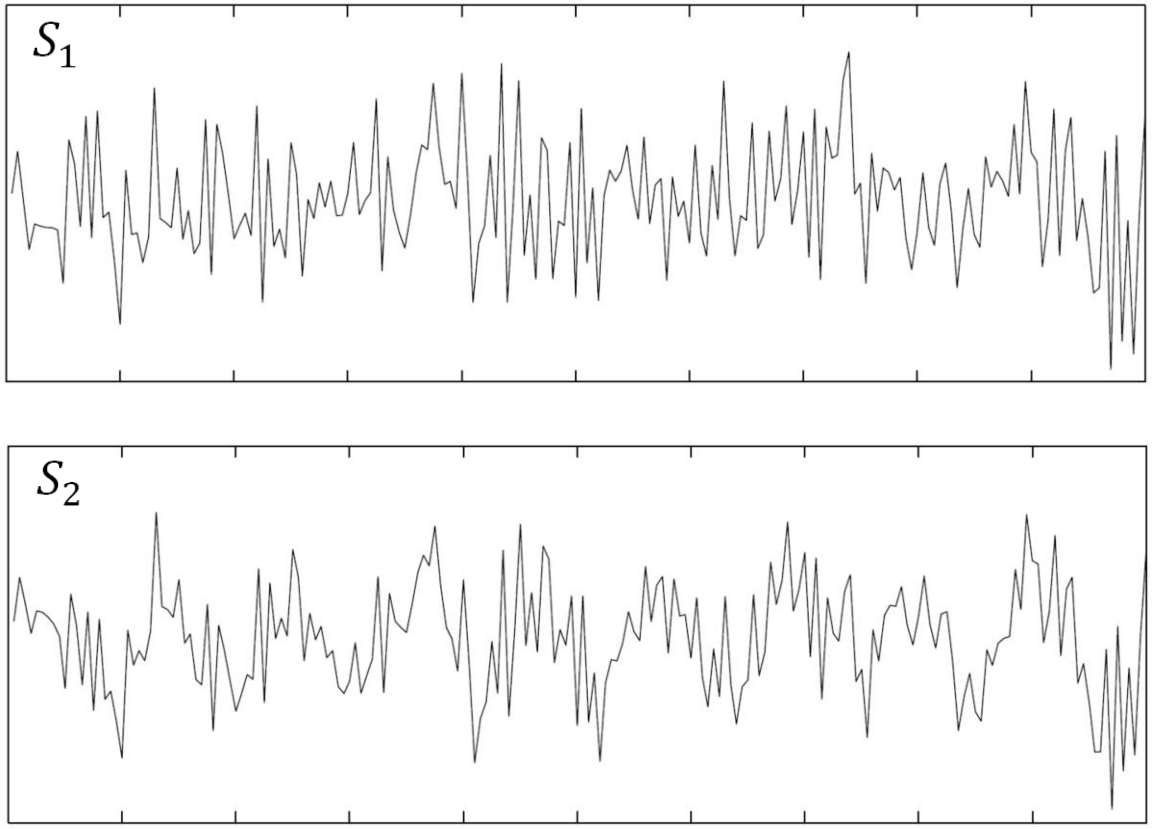
$$\mathbf{h}_1 = \Phi\mathbf{b}$$

- Compute DMD series

$$\hat{\mathbf{y}}_i(t) = \phi_i \exp(\omega_i t) b_i$$

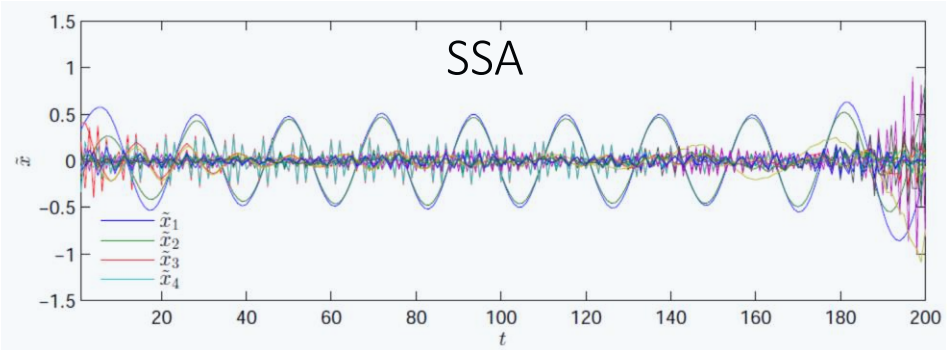
- Converting into $\tilde{\mathbf{y}}_i$

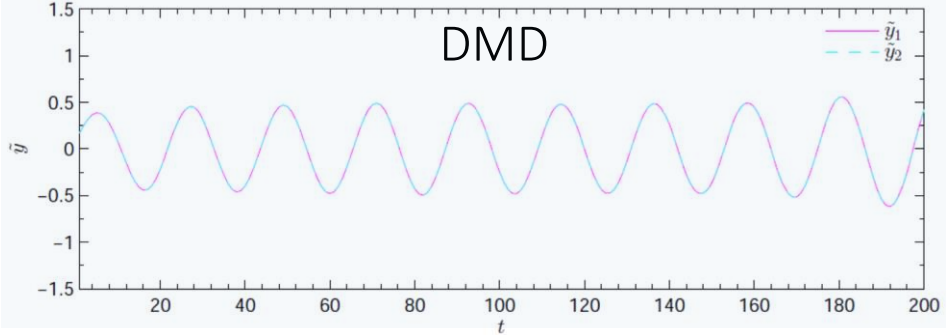
Detection of Structural Signal



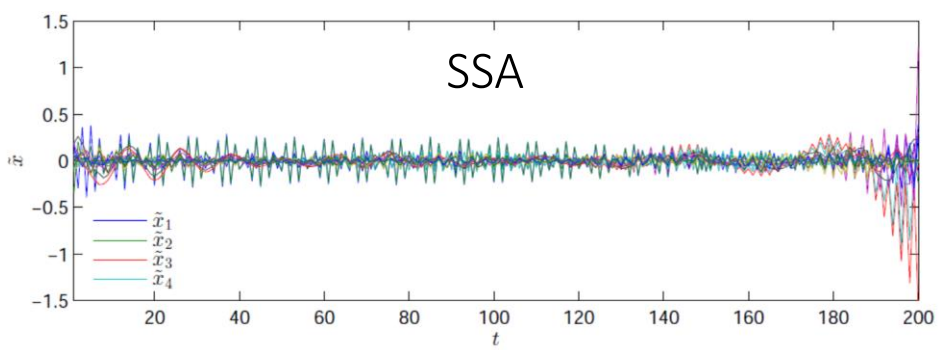
Detection of Structural Signal

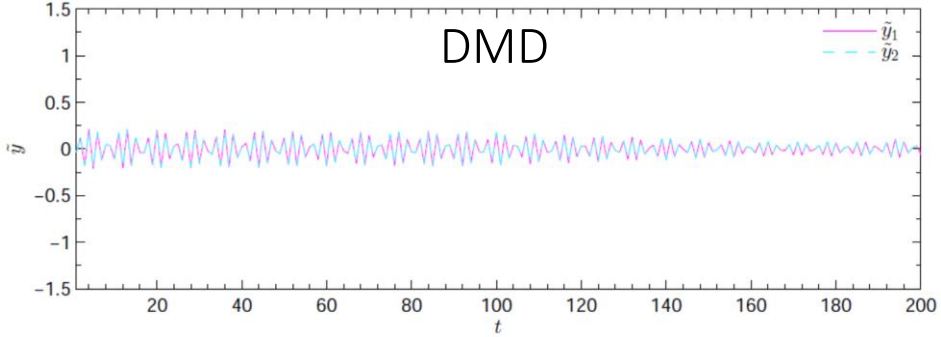
S_2





S_1

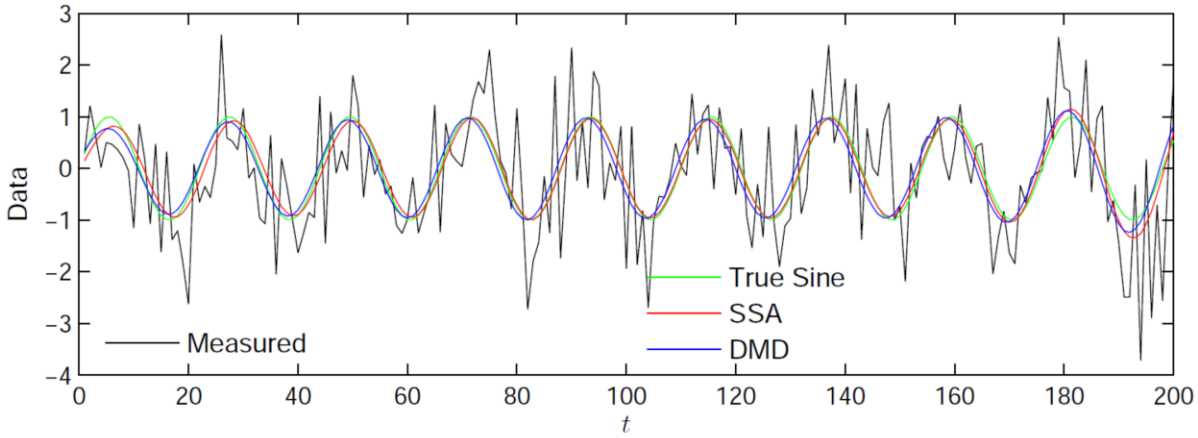




Detection of Structural Signal

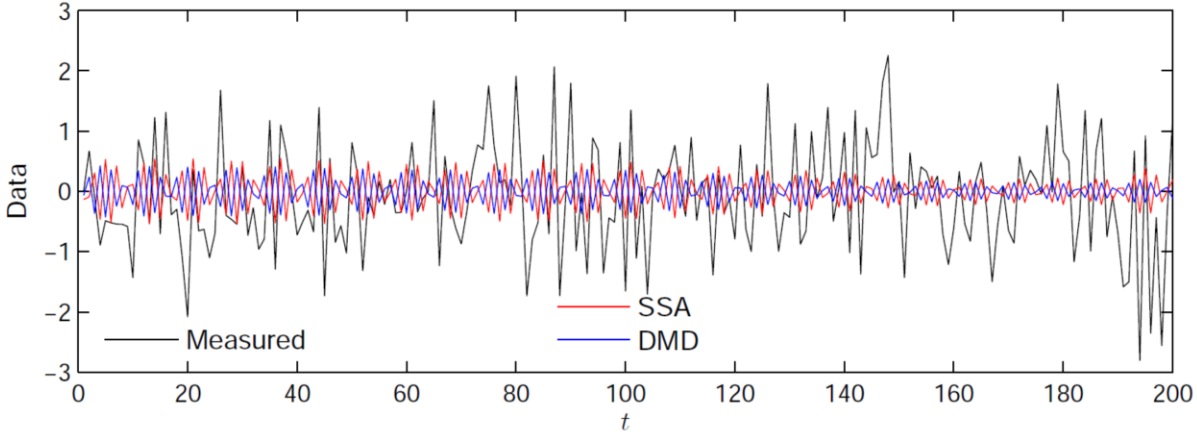
S_2

Reconstruction

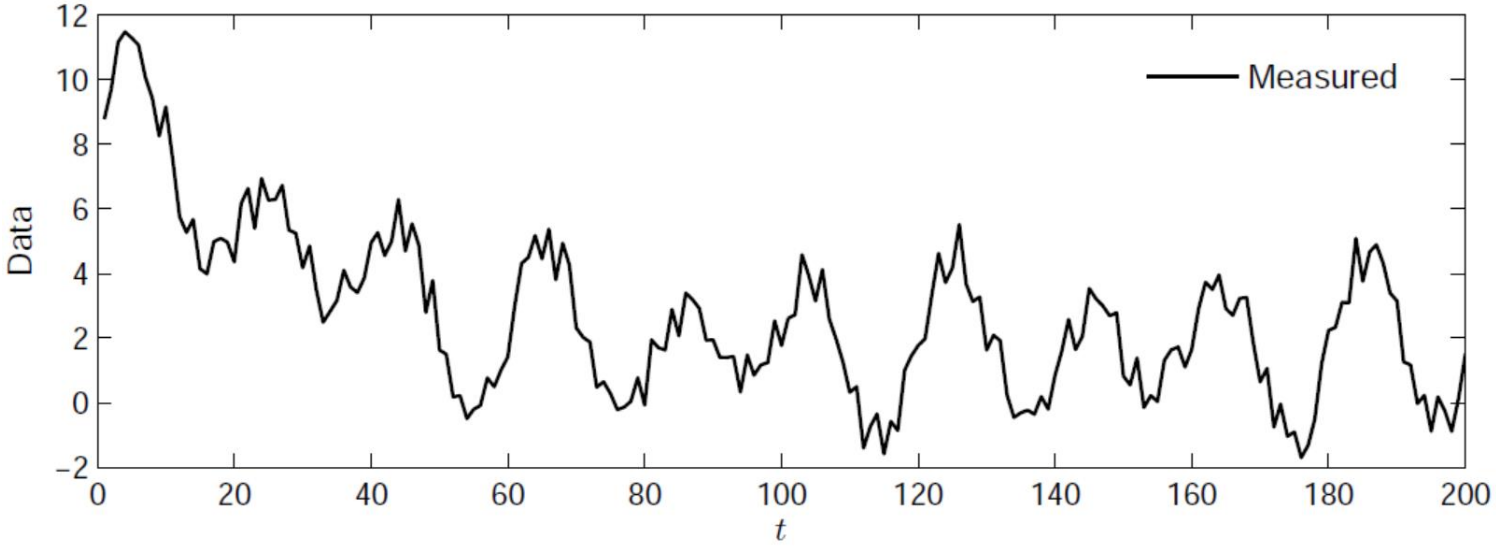


S_1

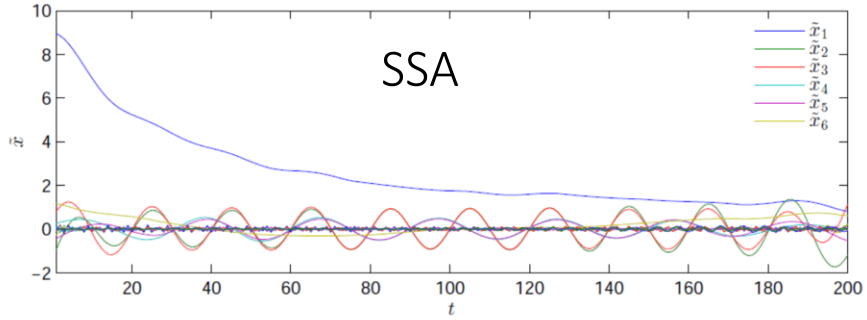
Reconstruction

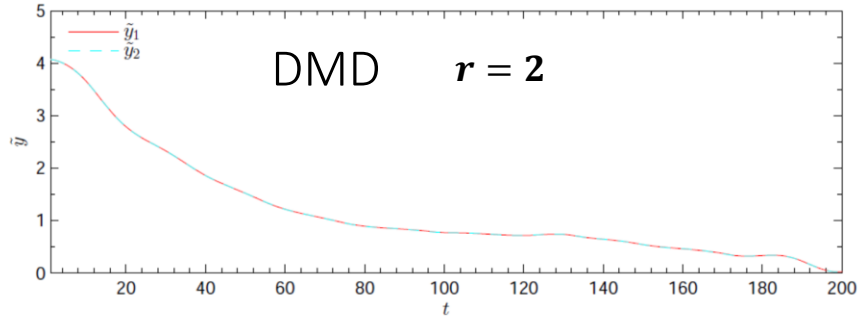


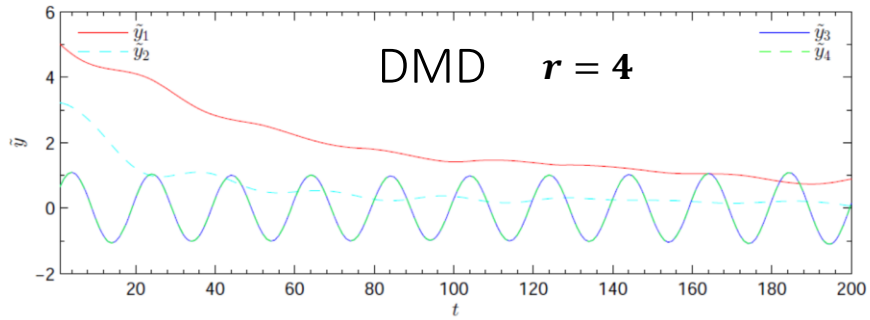
Extraction of Trend and Structures

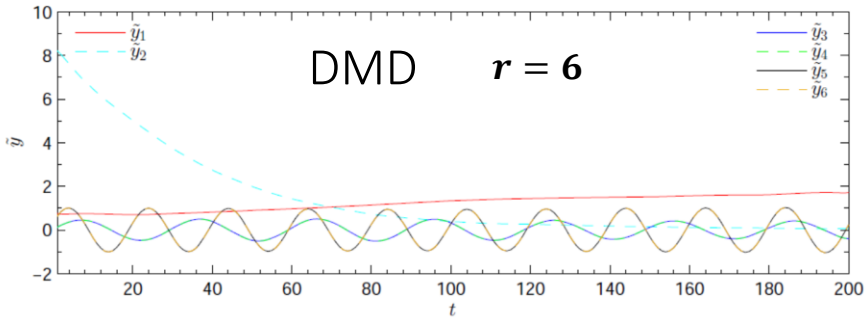


Extraction of Trend and Structures



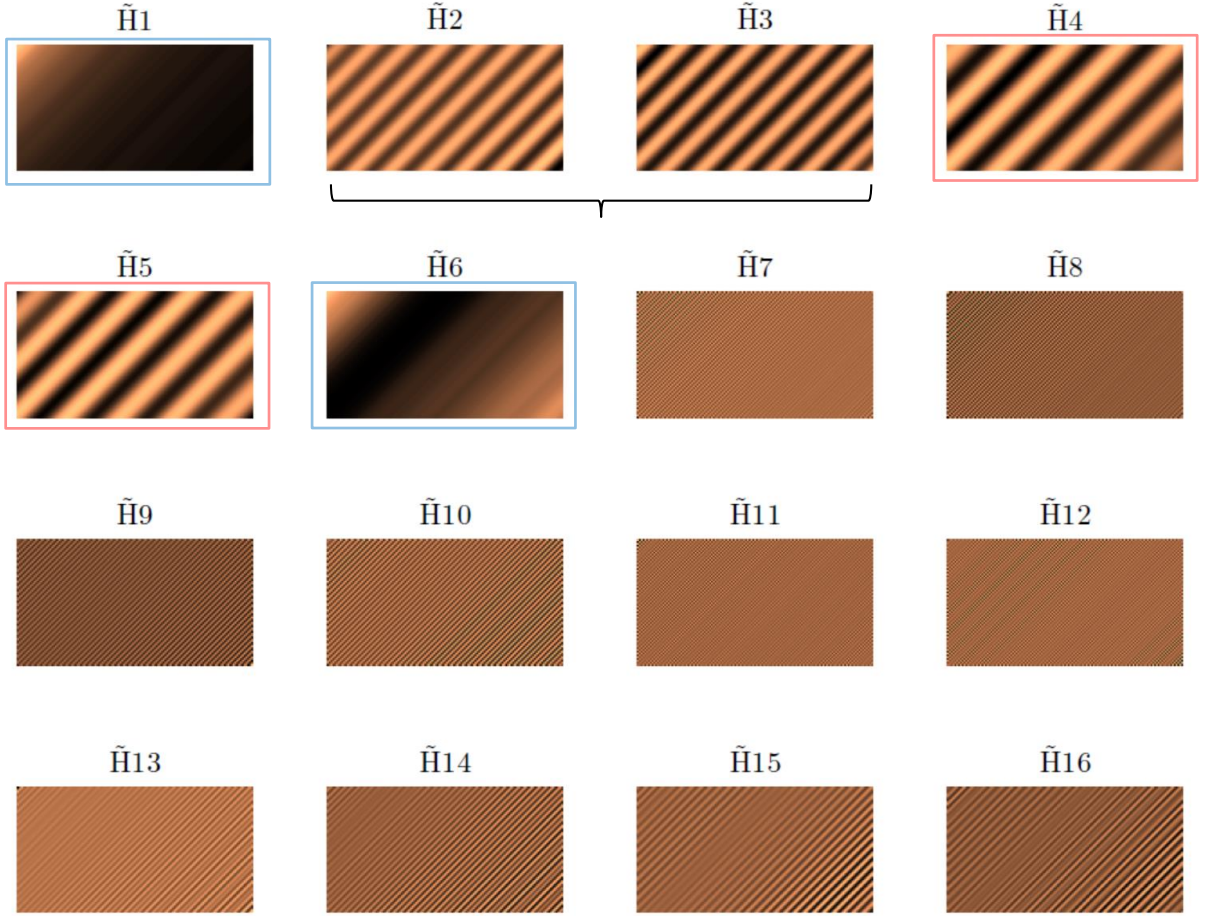






Extraction of Trend and Structures

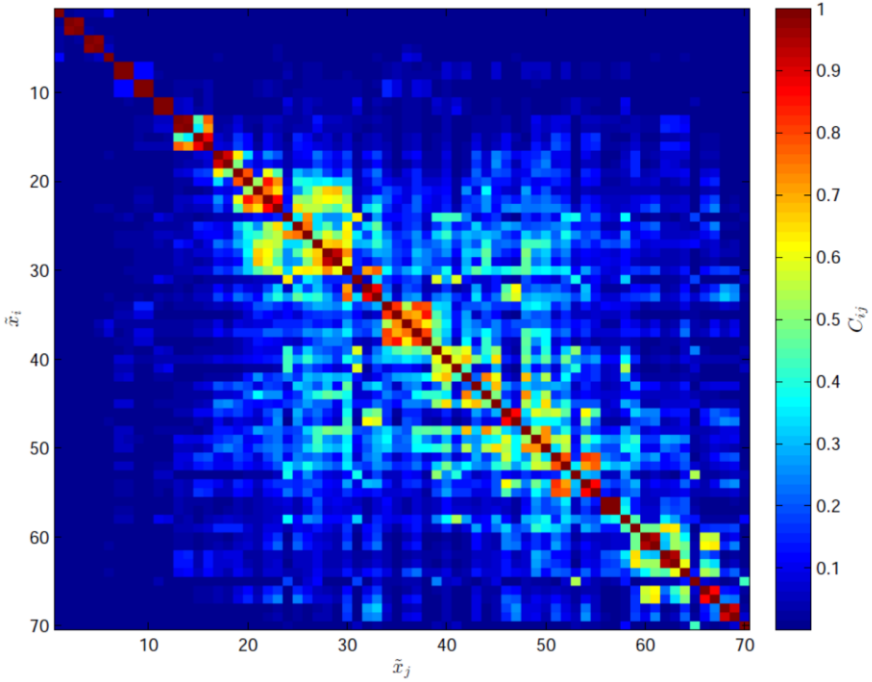
SSA Hankelized Elementary Matrices

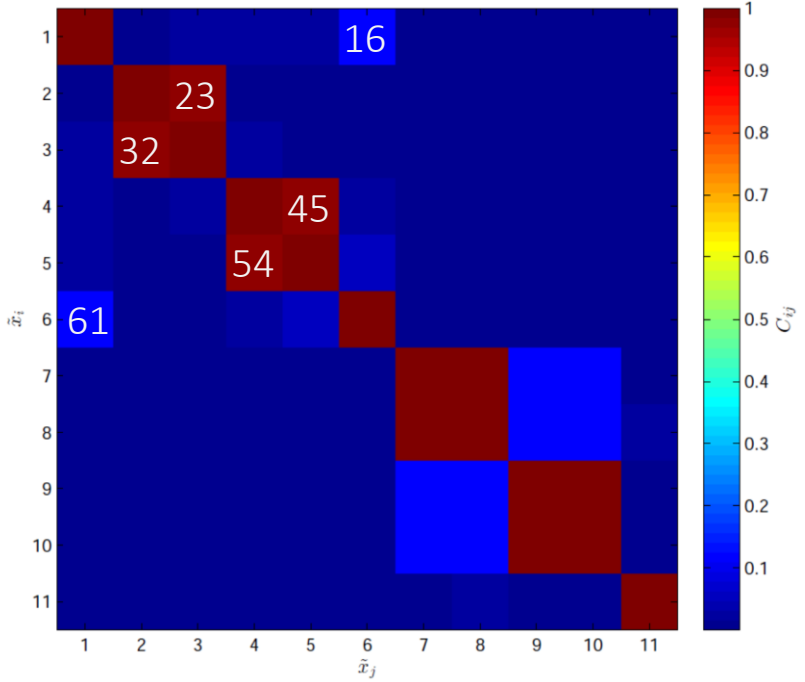


Extraction of Trend and Structures

Correlation map SSA components

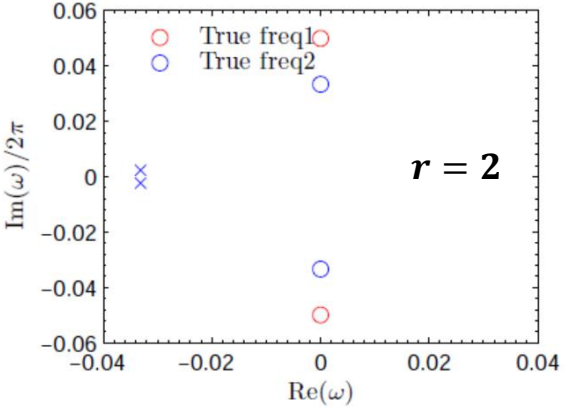
$$C_{ij} = \frac{\left(\tilde{\mathbf{x}}_i, \tilde{\mathbf{x}}_j\right)_w}{\|\tilde{\mathbf{x}}_i\|_w \left\|\tilde{\mathbf{x}}_j\right\|_w}$$

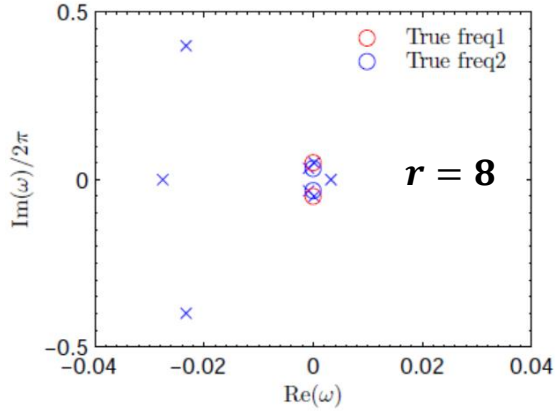




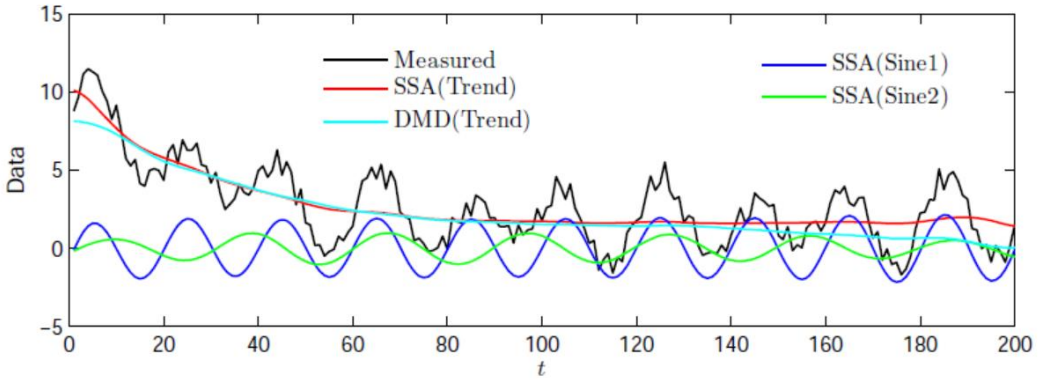
Extraction of Trend and Structures

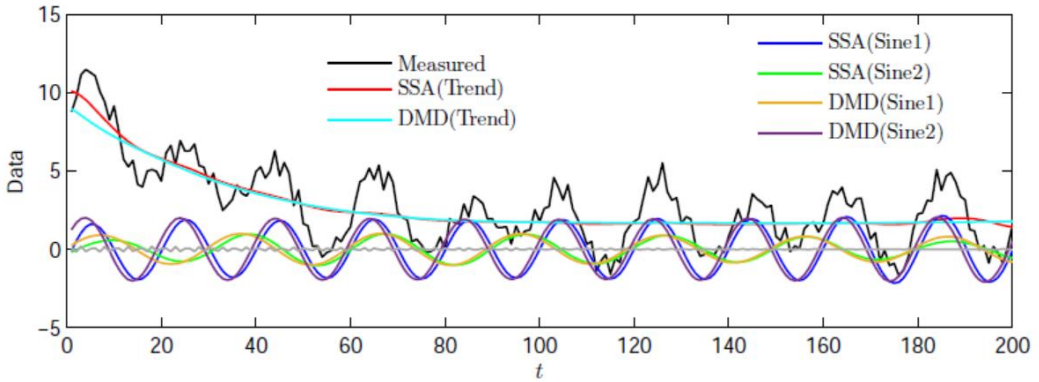
DMD frequency map



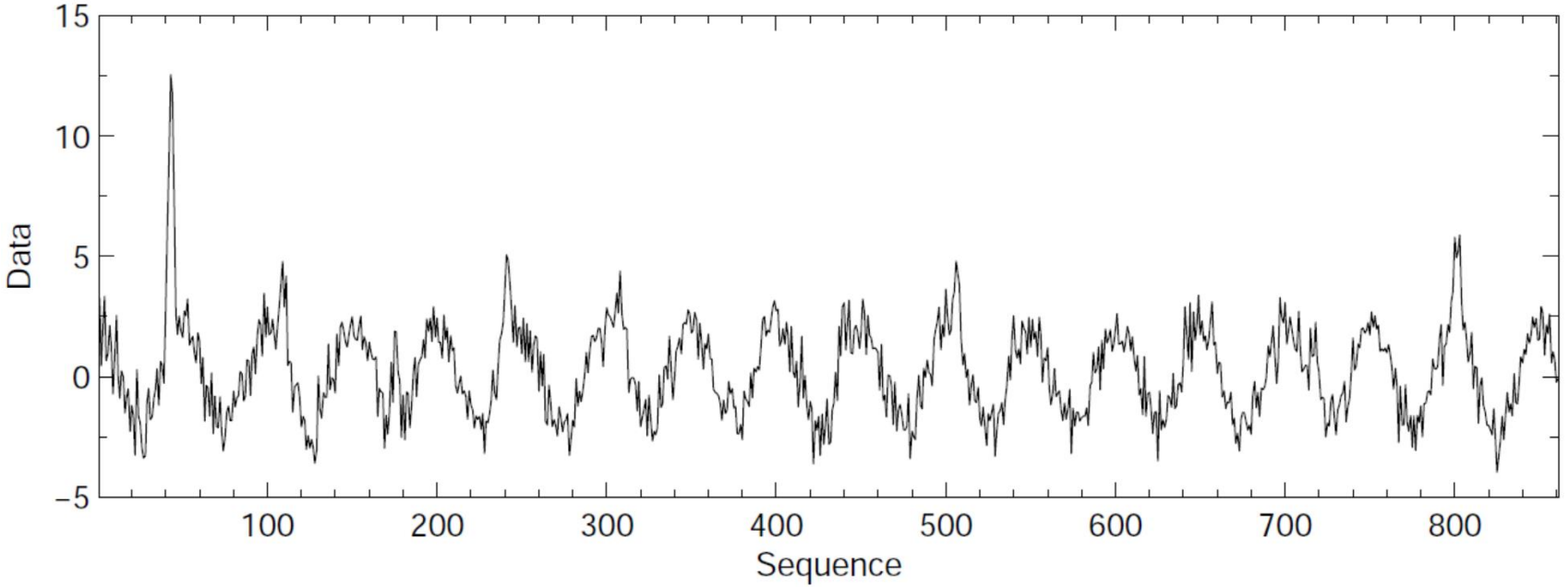


Extraction



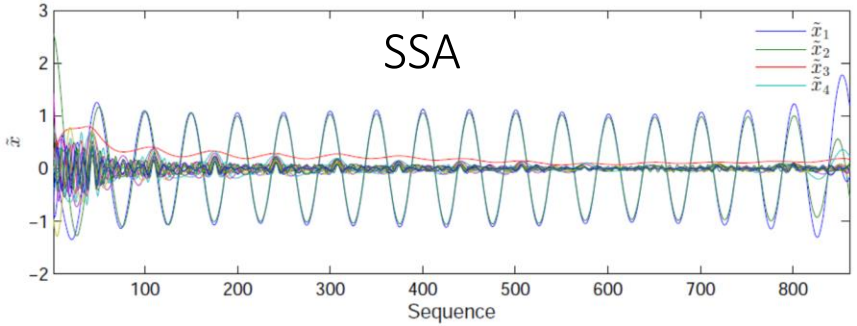


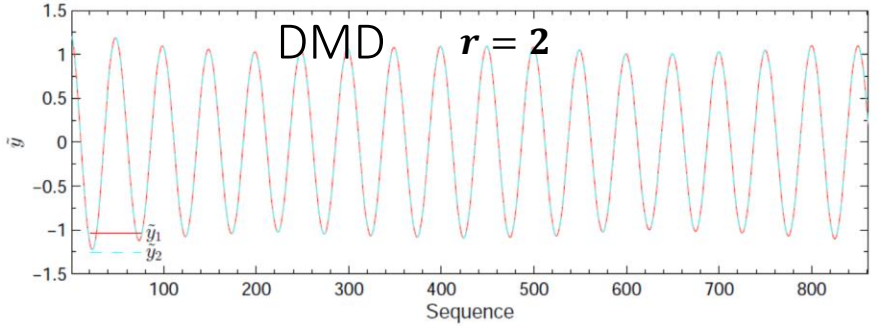
Separation of TEM signal from structure interference

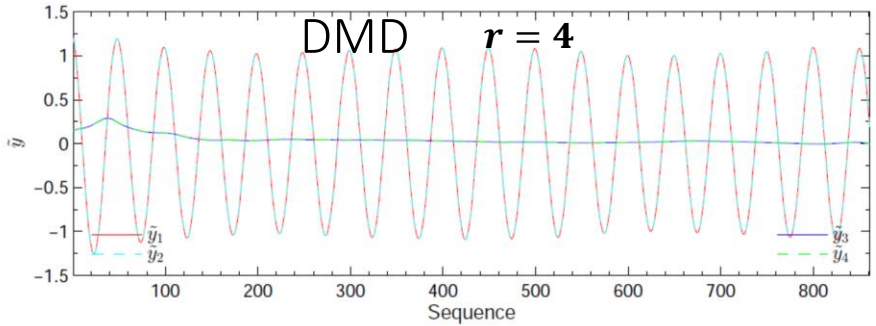


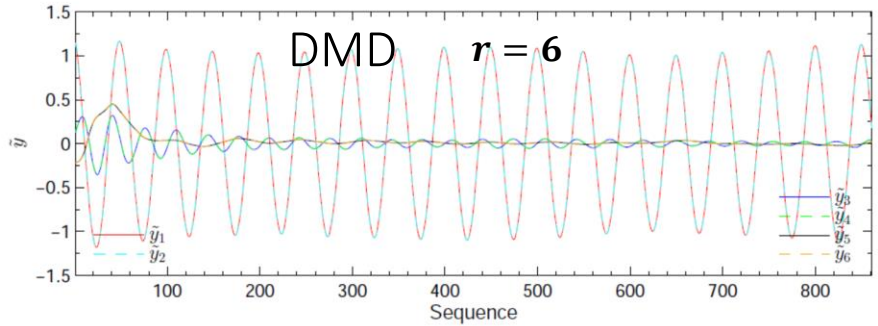
Separation of TEM signal from structure interference

SSA and DMD Components



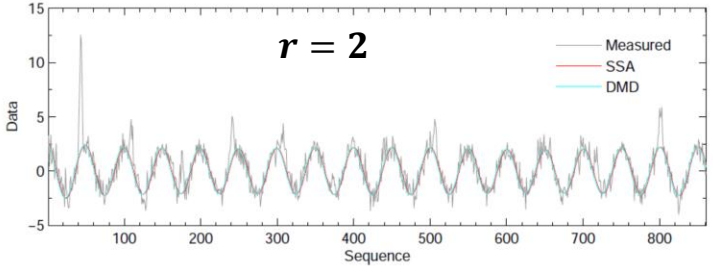


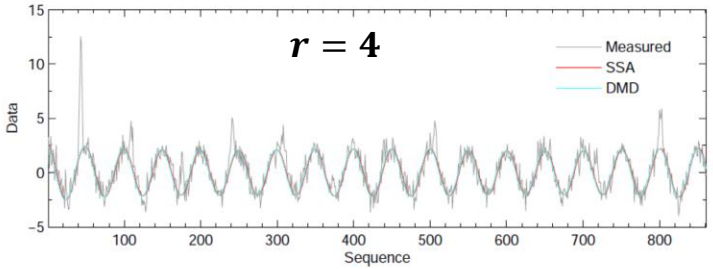


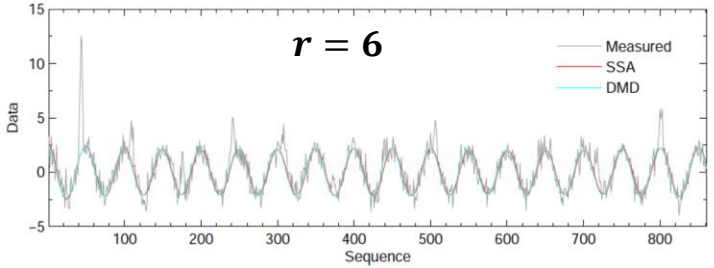


Separation of TEM signal from structure interference

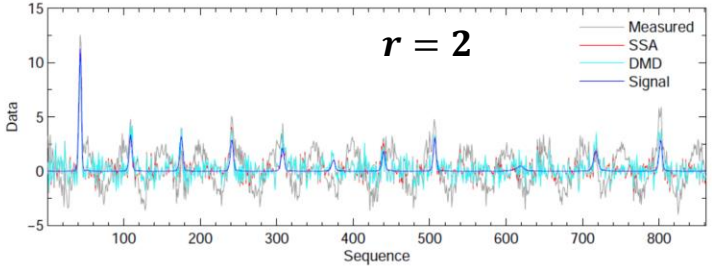
Synthesis

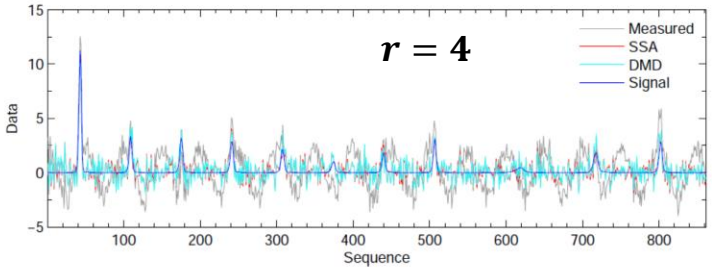


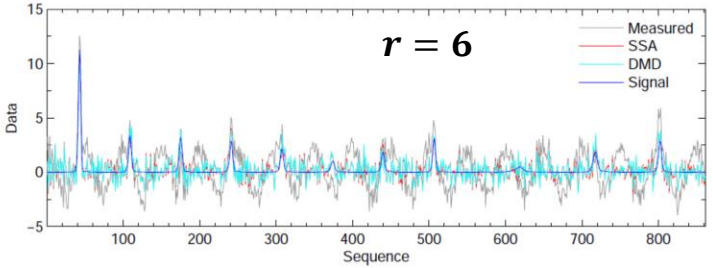




Separation

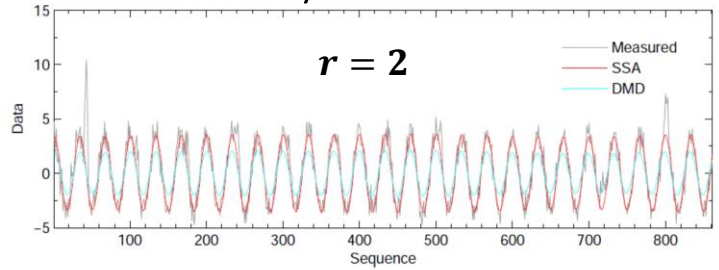


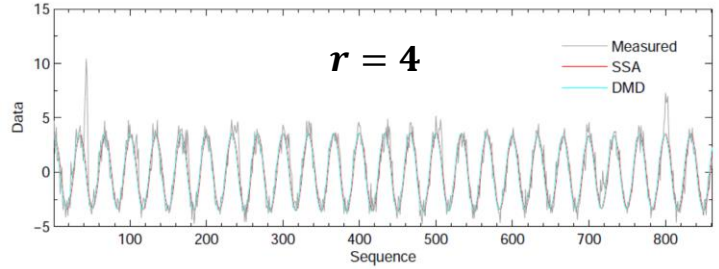


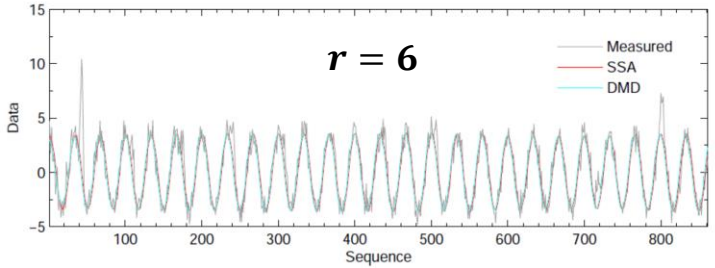


Separation of TEM signal from **strong** structure interference

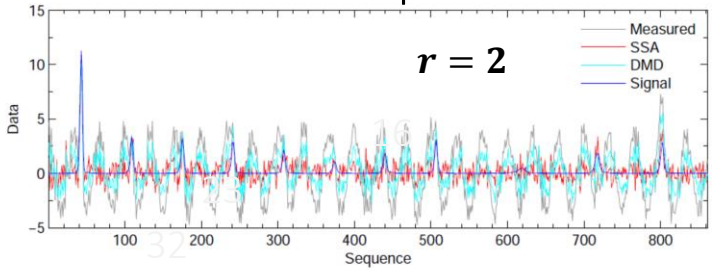
Synthesis

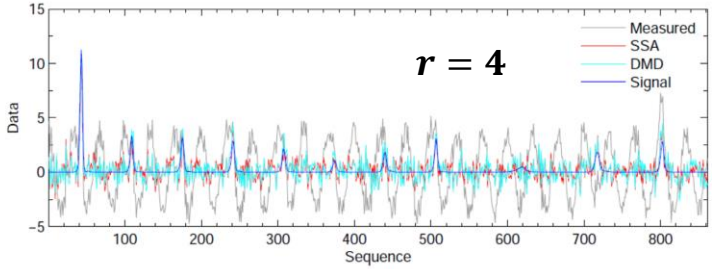


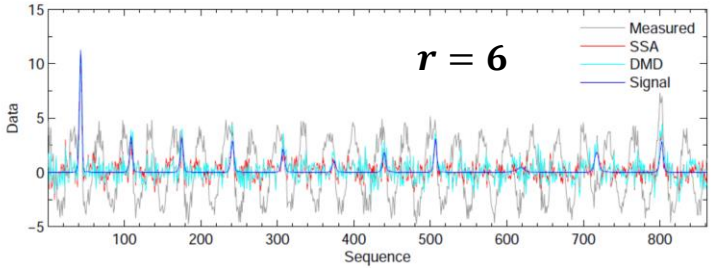




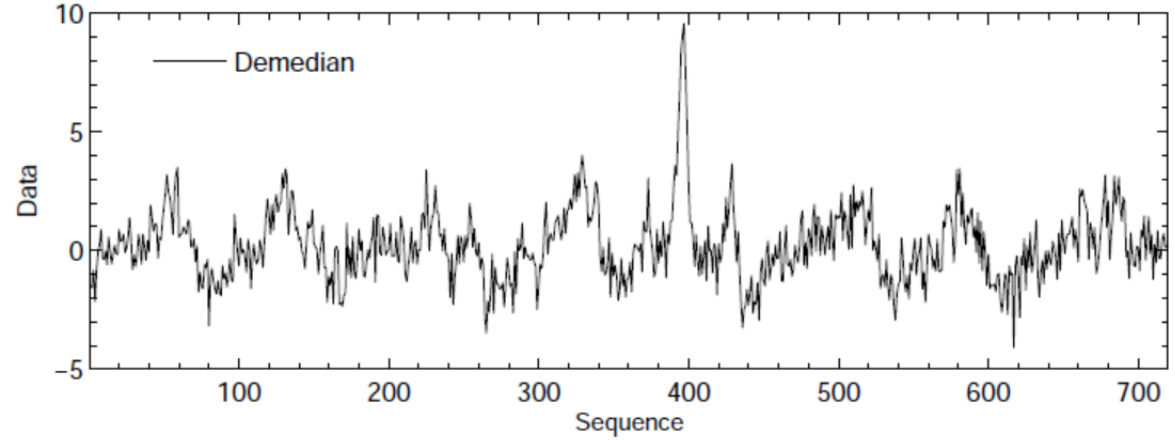
Separation

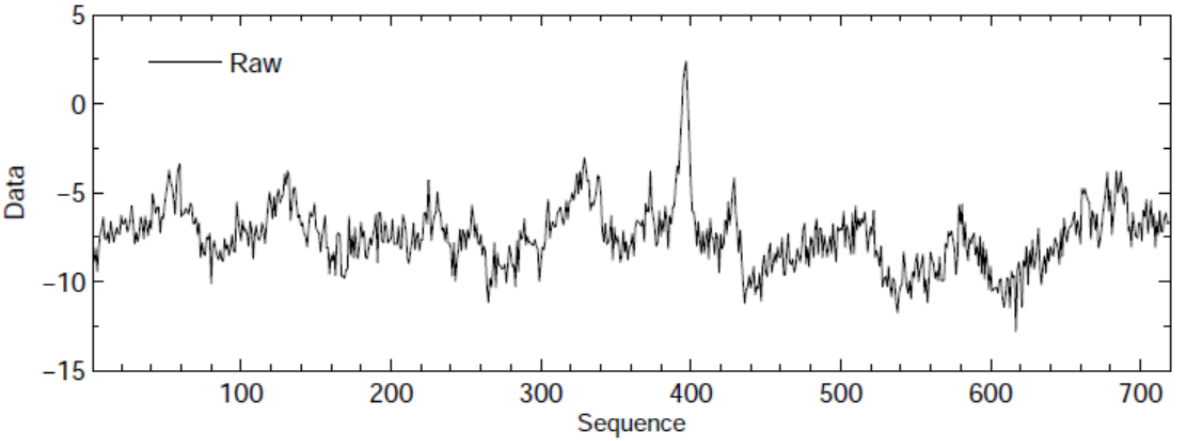






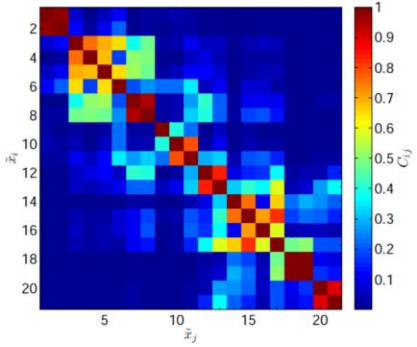
Application of SSA and DMD to noisy marine data

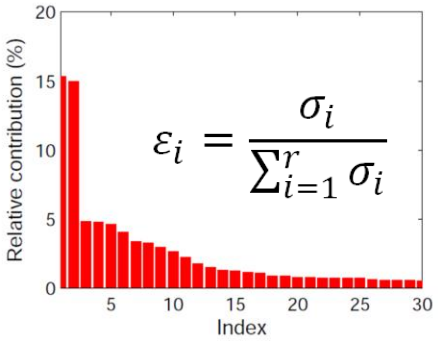




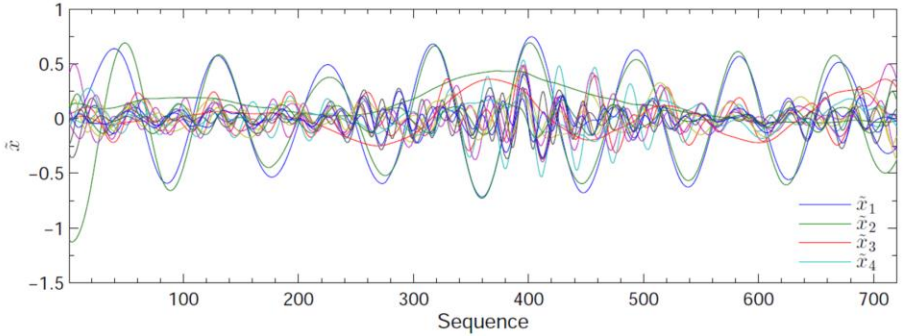
Application of SSA and DMD to noisy marine data

Demedian data: SSA

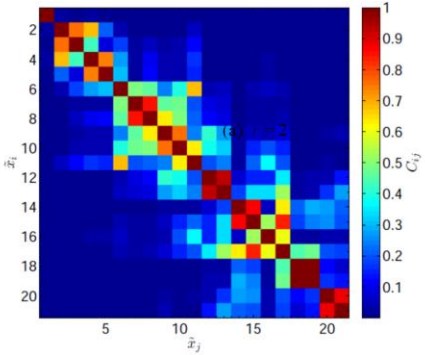


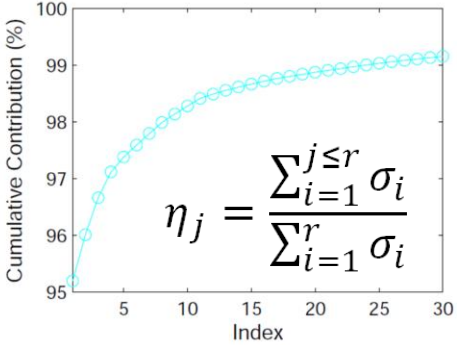


$$\varepsilon_i = \frac{\sigma_i}{\sum_{i=1}^{r} \sigma_i}$$

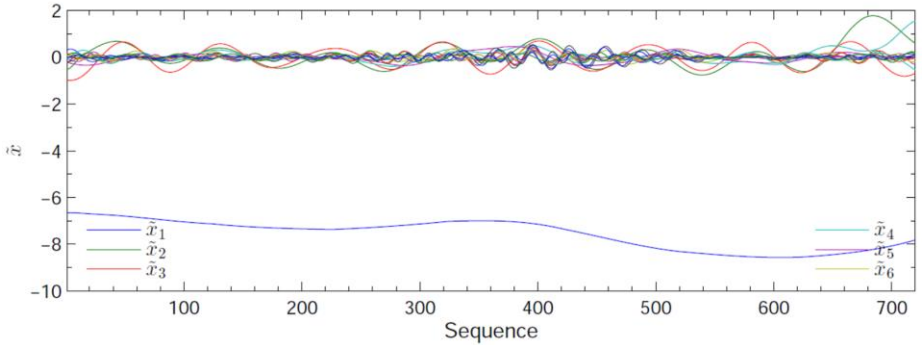


Raw data: SSA



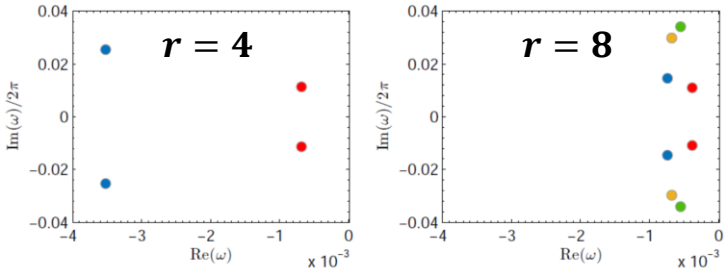


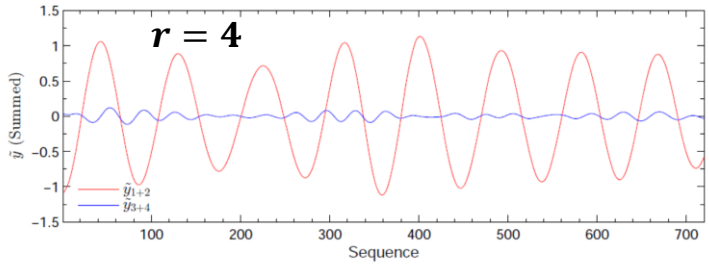
$$\eta_j = \frac{\sum_{i=1}^{j \le r} \sigma_i}{\sum_{i=1}^{r} \sigma_i}$$

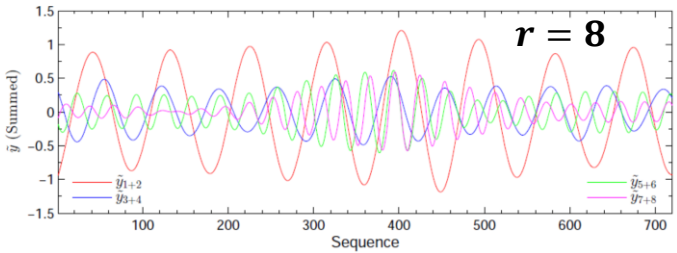


Application of SSA and DMD to noisy marine data

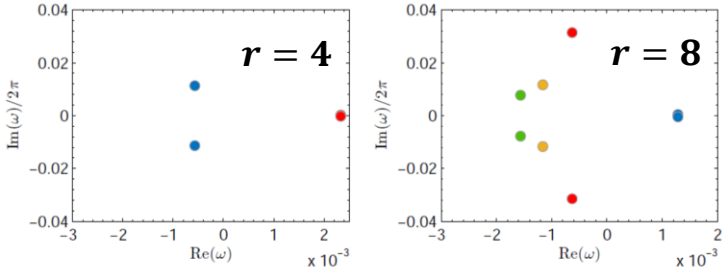
Demedian data: DMD

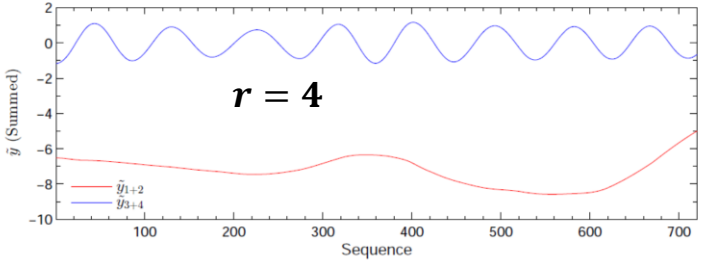


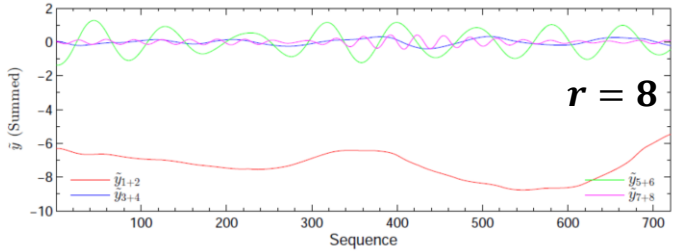




Raw data: DMD

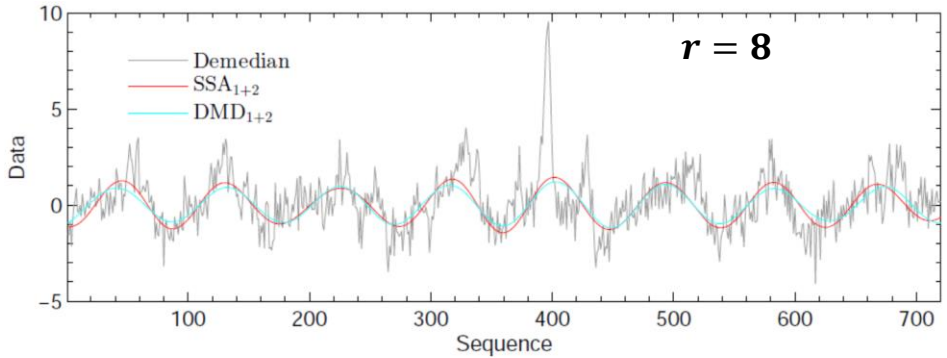


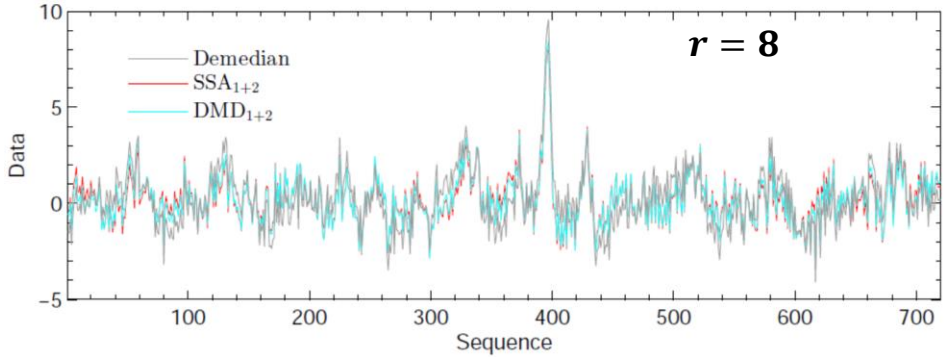




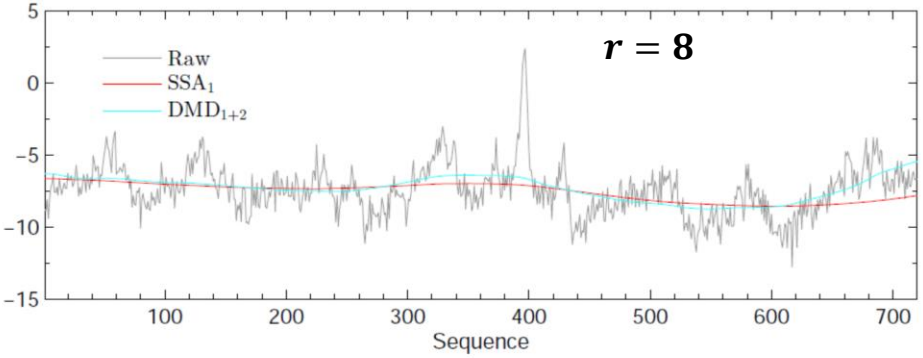
Application of SSA and DMD to noisy marine data

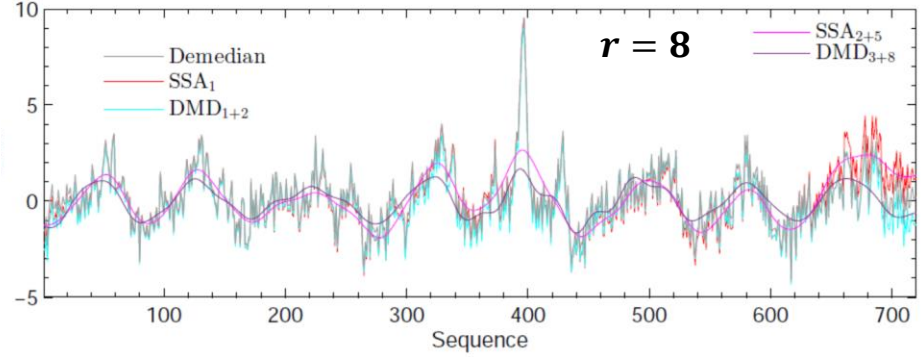
Demedian data: SSA and DMD





Raw data: SSA and DMD





Conclusion

- SSA and DMD techniques aims to decomposes a series into the sum of independent and interpretable components such as a slowly varying trend, oscillations, and a structure-less noise.

- The experiments show that both methods complement each other and are able to consistently capture trends and periodic evolution with frequencies in a series.

- The results demonstrate that, without any prior assumptions, both methods are promising for extracting inherent, hidden dynamics in a noisy response profile and isolating the desired components.

Acknowledgements

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